

Higher-Order Nonlinear Schrödinger equation with derivative non-Kerr nonlinear terms: A model for sub-10fs pulse propagation

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We analytically solved the higher-order nonlinear Schrödinger (HNLS) equation with non-Kerr nonlinearity under some parametric conditions and investigated explicitly bright and dark solitary wave solutions. Periodic wave solutions are also presented. The functional form of the bright and dark solitons presented are different from fundamental known *sech(.)* and *tanh(.)* respectively. We have estimated theoretically the size of the derivative non-Kerr nonlinear coefficients of the HNLS equation that agreed the reality of the waveguide made of highly nonlinear optical materials, could be used as the model parameters for sub-10fs pulse propagation.

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Optical solitons have promising potential to become principal information carriers in telecommunication due to their capability of propagating long distance without attenuation and changing their shapes. Considerable attentions are being paid theoretically and experimentally to analyze the dynamics of optical solitons in optical waveguide. The waveguides used in the picosecond optical pulse propagation in nonlinear optical communication systems are usually of Kerr type and consequently the dynamics of light pulses are described by nonlinear Schrödinger (NLS) family of equations with cubic nonlinear terms [1]. The validity of the NLS equation as a reliable model is dependent on the assumption that the spacial width of the soliton is much larger than the carrier wavelength. This is equivalent to the condition that the width of the soliton frequency spectrum is much less than the carrier frequency. The robustness of the optical soliton makes it useful for long distance optical communication systems, the high frequency of the optical carrier makes possible a high bit rate, and to increase the bit rate further it is desirable to use shorter femtosecond pulses and in order to model the propagation of a femtosecond (<100fs) optical pulse in an optical fiber, higher order nonlinear Schrödinger equation (HNLS) (not including optical fiber loss) [2]

$$E_z = i(a_1 E_{tt} + a_2 |E|^2 E) + a_3 E_{ttt} + a_4 (|E|^2 E)_t + a_5 E (|E|^2)_t \quad (1)$$

is required. Here z is the normalized distance along the fiber, t is the normalized time with the frame of the reference moving along the fiber at the group velocity. The subscripts z and t denotes the spatial and temporal partial derivatives respectively. The coefficients a_i ($i = 1, 2, \dots, 5$), particularly, ($a_1 = \frac{\beta_2}{2}$, $a_2 = \gamma_1$, $a_3 = \frac{\beta_3}{6}$, $a_4 = -\frac{\gamma_1}{\omega_0}$ and $a_5 = \gamma_1 T_R$) are the real parameters related to group velocity dispersion (GVD), self phase

modulation (SPM), third-order dispersion (TOD), self steepening and self-frequency shift due to stimulated Raman scattering (SRS) respectively. Here $\beta_j = (\frac{d^j \beta}{d\omega^j})_{\omega=\omega_0}$ is the dispersion coefficients evaluated at the carrier frequency ω_0 , with β_1 , the inverse of group velocity, β_2 , the group velocity dispersion parameter, β_3 third order dispersion (TOD) parameter and so on. β is propagation constant. More specifically, γ_1 is coefficient of cubic nonlinearity, which results from the intensity dependent refractive index. The term related to $\frac{\gamma_1}{\omega_0}$ results from the intensity dependence of the group velocity and causes self steepening and shock formation at the pulse edge. The last term related to $a_5 = \gamma_1 T_R$ incorporates the intrapulse Raman scattering and originates from the delayed response, which cause a self-frequency shift, where T_R , is called Raman time constant, can be estimated from the slope of the Raman gain (SRS). The characteristic Raman time constant T_R is defined as the first moment of the nonlinear response function [3]. Actually, to model intrapulse Raman Scattering the last coefficient a_5 should be $a_5 = i\gamma_1 T_R$. However, in the present work we succeeded in deriving analytical solutions in the case when a_5 was real. Let us remark that the latter case (a_5 real) also dominates in the analytical studies (e.g., Painlevé property, inverse scattering transform, Hirota direct method, conservation laws) undertaken to date [4] to show its integrable nature and obtained different types of exact solutions such as new solitary wave solution, w-shaped solution, bright and dark optical solitary wave solutions etc. Thus besides considering that the present works [4] realizes a significant advance as compared to previous literature, extension of the work to the case of a_5 imaginary represents a theoretical challenge that should be undertaken in the near future.

Present days applications in telecommunication and ultrafast signal routing systems, as the intensity of the incident light field becomes stronger, non-Kerr nonlinear effects come into play and due to these additional effects, the physical features and the stability of the NLS soliton

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can change. The way through the non-Kerr nonlinearity influences NLS soliton propagation is described by the NLS family of equations with higher degree of nonlinear terms [5]. To increase the channel handling capacity and ultra high speed pulse, it is necessary to transmit solitary waves at a high bit rate ($\approx 1 - 10$ fs) of ultrashort pulses, which can be seen in many applicative contexts such as high repetition pulse sources based on fiber technology [6]. At the same time, it is also important to include some additional higher-order perturbation effects to HNLS equation to analyze the solitary wave solution in non-Kerr nonlinear medium.

Here, in this Letter, we consider the higher-order NLS (HNLS) equation with non-Kerr term [7], can be written in terms of slowly varying complex envelope of the electric field $E(z, t)$, as

$$E_z = i(a_1 E_{tt} + a_2 |E|^2 E) + a_3 E_{ttt} + a_4 (|E|^2 E)_t + a_5 E (|E|^2)_t + ia_6 |E|^4 E + a_7 (|E|^4 E)_t + a_8 E (|E|^4)_t. \quad (2)$$

The terms related to coefficients a_6, a_7, a_8 in Eq. (2) represent the quintic non-Kerr nonlinearities. The quintic nonlinearities arise from the expansion of the refractive index in power of intensity I of the light pulse : $n = n_0 + n_2 I + n_4 I^2 + \dots$. Here n_0 is the linear refractive index coefficient and n_2, n_4 are the nonlinear refractive index coefficients, originate from third- and fifth-order susceptibilities respectively. The polarizations induced through these susceptibilities give the cubic and quintic (non-Kerr) terms in nonlinear Schrödinger equation respectively. The nonlinearity arises due to fifth-order susceptibility can be obtained in many optical materials such as semiconductors, semiconductor doped glasses, Al-GaAs, polydiacetylene toluene sulfonate (PTS), chalcogenide glasses and some transparent organic materials. When the last three terms related to a_6, a_7, a_8 of Eq. (2) are ignored, the resulting equation becomes the HNLS equation as given in Eq. (1). In a recent paper [7] we have investigated the Dark-in-the-Bright (DITB) solitary wave solution of Eq. (2). The DS or DITB solitary wave solution is composed of the product of bright and dark solitary waves. We also investigated the stability of the DITB solution under some initial perturbation on the parametric conditions. The shape of pulse remains unchanged up to 20 normalized length even under some very small violation in parametric conditions. More recently [8], we have studied modulational instability (MI) of Eq. (2) with forth-order dispersion in context of optics and presented an analytical expression for MI gain to show the effects of non-Kerr nonlinearities and higher-order dispersions on MI gain spectra. In our study we also demonstrate that MI can exist not only for anomalous group velocity dispersion (GVD) regime but also in the normal GVD regime and also investigated that the quintic non-Kerr nonlinear terms are more important over the cubic Kerr nonlinearity because non-Kerr nonlinearities are responsible for stability of localized solutions. But in the previous works [5, 7, 8] the solitary wave solutions in presence of higher non-Kerr nonlinearity have not been

investigated. In a very recent work, Triki and Taha [9] presented solitary wave solutions for HNLS equation including non-Kerr nonlinear terms upto the coefficient a_6 of Eq. (2). In this article, we shall study the bright- and dark solitary wave solutions for HNLS equation that contains time derivative of non-Kerr nonlinear terms and estimate the size of model coefficients of Eq. (2) which will be useful for propagation of very short pulse of width around sub-10fs in highly nonlinear optical fibers.

To investigate the existence of analytic wave solution of HNLS equation in presence of non-Kerr terms we begin by scaling the variables of the Eq. (2) in the form

$$E = b_1 \Psi, \quad z = b_2 \xi, \quad \text{and} \quad t = b_3 \tau$$

and choosing b_1, b_2 and b_3 such that the coefficients corresponding to GVD, SPM and TOD become unity. Thus in the scaled formed of the Eq. (2) becomes

$$\Psi_\xi = i(\Psi_{\tau\tau} + |\Psi|^2 \Psi) + \Psi_{\tau\tau\tau} + \alpha_1 (|\Psi|^2 \Psi)_\tau + \alpha_2 \Psi (|\Psi|^2)_\tau + i\alpha_3 |\Psi|^4 \Psi + \alpha_4 (|\Psi|^4 \Psi)_\tau + \alpha_5 \Psi (|\Psi|^4)_\tau \quad (3)$$

where

$$\alpha_1 = \frac{b_1^2 b_2 a_4}{b_3} = \frac{a_4 a_1}{a_2 a_3}, \quad \alpha_2 = \frac{b_1^2 b_2 a_5}{b_3} = \frac{a_5 a_1}{a_2 a_3},$$

$$\alpha_3 = b_1^4 b_2 a_6 = \frac{a_6 a_1^3}{a_2^2 a_3^2}, \quad \alpha_4 = \frac{b_1^4 b_2 a_7}{b_3} = \frac{a_7 a_1^4}{a_2^2 a_3^3} \text{ and}$$

$$\alpha_5 = \frac{b_1^4 b_2 a_8}{b_3} = \frac{a_8 a_1^4}{a_2^2 a_3^3}.$$

In writing (3) we have chosen $b_1 = \left(\frac{a_3}{a_2^2 a_3^2}\right)^{\frac{1}{2}}$, $b_2 = \frac{a_2}{a_1}$ and $b_3 = \frac{a_3}{a_1}$.

To obtain the exact solitary wave solutions of Eq.(3) we consider a solution of the following form

$$\Psi(\xi, \tau) = \mathcal{P}(\tau + v\xi) e^{i(k\xi - \Omega\tau)} = \mathcal{P}(\chi) e^{i(k\xi - \Omega\tau)}, \quad (4)$$

with $\mathcal{P}(\chi)$ real. On substitution Eq. (4) into Eq. (3) and after removing the exponential terms one can obtain the real and imaginary parts of the resulting equation as

$$\mathcal{P}_{\chi\chi} = (v - 2\Omega + 3\Omega^2) \mathcal{P} - \frac{3\alpha_1 + 2\alpha_2}{3} \mathcal{P}^3 - \frac{5\alpha_4 + 4\alpha_5}{5} \mathcal{P}^5 \quad (4a)$$

and

$$\mathcal{P}_{\chi\chi} = \frac{(k + \Omega^2 - \Omega^3)}{(1 - 3\Omega)} \mathcal{P} - \frac{(1 - \Omega\alpha_1)}{(1 - 3\Omega)} \mathcal{P}^3 - \frac{(\alpha_3 - \Omega\alpha_4)}{(1 - 3\Omega)} \mathcal{P}^5. \quad (4b)$$

In the following we will discuss two cases:

Case 1: $\Omega \neq \frac{1}{3}$. Equating these two equations we get the following necessary and sufficient conditions on Ω and equation for k in terms of Ω :

$$\Omega = \frac{3\alpha_1 + 2\alpha_2 - 3}{6(\alpha_1 + \alpha_2)} = \frac{5\alpha_4 + 4\alpha_5 - 5\alpha_3}{(10\alpha_4 + 12\alpha_5)} \quad (5a)$$

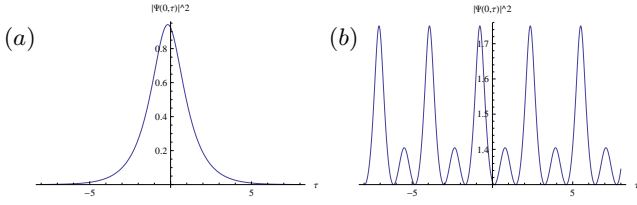


FIG. 1: (Color online) (a) Intensity of the solitary wave profile $|\Psi(0, \tau)|^2$ as a function of τ for the value $\alpha_1 = \alpha_2 = 1$ and $v = 1$ (b) Periodic wave profile $|\Psi(0, \tau)|$ as a function of τ as computed from Eq. (10) for the value $\alpha_1 = \alpha_2 = 1$ and $v = -1$.

and

$$k = (1 - 3\Omega)(v - 2\Omega + 3\Omega^2) - \Omega^2 + \Omega^3. \quad (5b)$$

with constraint relations

$$\alpha_4 = \frac{3\alpha_1}{5}, \quad \alpha_5 = \frac{\alpha_2}{2} \quad \text{and} \quad \alpha_3 = \frac{3}{5}. \quad (6)$$

The function $\mathcal{P}(\chi)$ satisfies the ordinary nonlinear differential equation in (4a). Multiplying (4a) by \mathcal{P}_χ and integrating we get

$$(\mathcal{P}_\chi)^2 = a\mathcal{P}^2 - b\mathcal{P}^4 - c\mathcal{P}^6 + 2\mathcal{E}, \quad (7)$$

where $(v - 2\Omega + 3\Omega^2) = a$, $\frac{3\alpha_1 + 2\alpha_2}{6} = b$, $\frac{5\alpha_4 + 4\alpha_5}{15} = c$ and \mathcal{E} is the arbitrary constant of integration. Eq. (7) describe the evolution of the anharmonic oscillator with potential

$$U(\mathcal{P}) = -\frac{a}{2}\mathcal{P}^2 + \frac{b}{2}\mathcal{P}^4 + \frac{c}{2}\mathcal{P}^6 \quad (8)$$

and \mathcal{E} , the integration constant in (7) is the energy of that anharmonic oscillator. For zero energy ($\mathcal{E} = 0$) we have find the solution of Eq.(7) as

$$\mathcal{P}(\chi) = \frac{2a^{\frac{3}{4}} \sqrt{(\sqrt{a} - b e^{2\sqrt{a}\chi})^2 + 4ac e^{4\sqrt{a}\chi}} e^{\sqrt{a}\chi}}{\sqrt{((a - (b^2 - 4ac) e^{4\sqrt{a}\chi})^2 + 16ab^2 c e^{8\sqrt{a}\chi})}}, \quad (9)$$

provided that $a > 0$, $b > 0$. Now using Eqs.(4), (5b) and (9) we can write the solution of Eq. (3) as

$$\Psi(\xi, \tau) = \mathcal{P}(\tau + v\xi) e^{i((1-3\Omega)(v-2\Omega+3\Omega^2)-\Omega^2+\Omega^3)\xi - \Omega\tau}. \quad (10)$$

The intensity profile of the solitary wave solution (Eq. (10)) is shown in Fig. 1(a), as computed from Eq. (3) for the values $\alpha_1 = \alpha_2 = 1$ and $v = 1$. One can check the evolution of the intensity profile and it is an interesting to note that the wave profile remains unchanged during the evolution. For negative value of the parameter a , Eq. (3) shows the periodic solution. We have presented in Fig. 1(b) the periodic wave solution. Here we have taken the same parameter values as that in Fig. 1(a) but $v = -1$ such that $a < 0$. For $\Omega = 0$ (i.e. $3\alpha_1 + 2\alpha_2 = 3$) and consequently, $a = v$, $b = \frac{1}{2}$ and $c = \frac{1}{5}$ one can verify the solitary wave solution and periodic solution of Eq. (3)

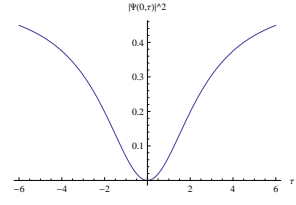


FIG. 2: (Color online) Intensity of the solitary wave profile $|\Psi(0, \tau)|^2$ (Eq.(12)) as a function of τ as computed from Eq. (3) for the value $\alpha_1 = \alpha_2 = 1$ and $v = -\frac{4}{9}$

from Eq. (10) for $a > 0$ and $a < 0$ respectively.

Dark solitary wave. We have seen for $\mathcal{E} = 0$, HNLS equation in presence of non-Kerr terms supports the bright optical wave solution provided that the parameter a in Eq. (7) is positive. Now we chose $\mathcal{E} = -\frac{a^2}{6b}$ and $c = -\frac{b^2}{3a}$ such that we can write Eq.(7) in the form

$$d\chi = \left(-\frac{(a - b\mathcal{P}^2)^3}{3ab} \right)^{-\frac{1}{2}} d\mathcal{P}. \quad (11)$$

Integrating Eq. (11) we find the solitary wave solution of Eq. (3), with the same constraint relations stated above in (6), as

$$\Psi(\xi, \tau) = \frac{a(\tau + v\xi)}{\sqrt{b}\sqrt{(-3 + a(\tau + v\xi)^2)}} e^{i(k\xi - \Omega\tau)}, \quad (12)$$

with $k = (1 - 3\Omega)(v - 2\Omega + 3\Omega^2) - \Omega^2 + \Omega^3$, provided $a < 0$, with $v = 2\Omega - 3\Omega^2 - \frac{5}{36}(3\alpha_1 + 2\alpha_2)$. In Fig. 3 we have plotted the intensity profile of the optical solitary wave solution with the parameter values $\alpha_1 = \alpha_2 = 1$ and $v = -\frac{4}{9}$. We call this solution as a dark solitary wave in the sense that the intensity profile associated with such soliton exhibits a dip in a uniform background and asymptotic absolute value of $\Psi(\xi, \tau)$ tends toward a constant nonzero value for large values of τ . In this context, although bright solitons are relatively easy to generate in optical fiber, the dark solitons are less sensitive to optical fiber loss, less influenced by noise and are more stable against Gordon Haus jitter in long communication line [10]. As the mutual interaction between two neighboring dark solitons is much weaker than that between two bright solitons [11], so the properties of the dark soliton attracted scientists very much in the communication systems. But it is difficult to use a dark soliton with a $\tanh(\cdot)$ -type wave form in a transmission system because dark pulse cannot be easily generated.

Case 2: $\Omega = \frac{1}{3}$, Eq. (4b) takes the form

$$(k + \Omega^2 - \Omega^3)\mathcal{P} - (1 - \Omega\alpha_1)\mathcal{P}^3 - (\alpha_3 - \Omega\alpha_4)\mathcal{P}^5 = 0. \quad (13)$$

Setting the coefficients of \mathcal{P} , \mathcal{P}^3 and \mathcal{P}^5 to zero in Eq. (13) and using $\Omega = \frac{1}{3}$ we obtain $k = -\frac{2}{27}$ and the con-

straint condition

$$\Omega = \frac{1}{\alpha_1} \quad \text{and} \quad \Omega = \frac{\alpha_3}{\alpha_4}, \text{ which implies } \alpha_4 = \alpha_1 \alpha_3. \quad (14)$$

In this case, $\mathcal{P}(\chi)$ satisfy the ordinary differential equation similar to that in (7) but only differs in the coefficient values.

$$(\mathcal{P}_\chi)^2 = a'\mathcal{P}^2 - b'\mathcal{P}^4 - c'\mathcal{P}^6 + 2\mathcal{E}'. \quad (15)$$

Here, in this case, $(v - \frac{1}{3}) = a'$, $\frac{9+2\alpha_2}{6} = b'$, $\frac{5\alpha_4+4\alpha_5}{15} = c'$ and \mathcal{E}' is the arbitrary constant of integration. In case 2, similar to Eq. (7) one can solve Eq. (15) to get the bright, dark and also periodic wave solutions of Eq. (3). Only here the coefficients a' , b' and c' are different from that in Eq. (7).

Before conclusion, let us discuss some application of the above theoretical prediction. For large channel handling capacity in the frame of dense time-domain multiplexing and for high speed, it is necessary to transmit solitary wave at a high bit rate of ultrashort pulses. At the same time, it is also important to consider the higher-order non-Kerr like nonlinearity including derivative in HNLS equation for sub-10fs pulse propagation. The relevance of these terms is also important in the frame of post-soliton compression that can be achieved in highly nonlinear material. Compared to silica glasses, as chalcogenide glasses exhibit an extremely high nonlinear refractive-index coefficient that can be two or three orders of magnitude greater than that of silica at 1.55 μm . They also offer several distinctive optical properties such as a transmission window that extends far into the infrared (IR) spectral region (up to 25 μm for telluride glasses). Because of high nonlinearity and large IR transparency, chalcogenide fibers are well suited for compact Raman amplifiers, supercontinuum generation and other mid-IR sources. For experimental verification of the propagation of solitary wave, one may use the waveguide made of chalcogenide glasses which are made from heavy chalcogen elements such as S, As, Se, Te, having electron shells which are easily polarizable under an electro-magnetic field excitation. For example, the chalcogenide glass As_2Se_3 , the nonlinear index coefficients [12, 13] are $n_2 = 2.2 \times 10^{-17} \text{ m}^2/\text{W}$ and $n_4 = -6.5 \times 10^{-31} \text{ m}^4/\text{W}^2$ and for the As_2S_3 sample, the values are $n_2 = 4.2 \times 10^{-18} \text{ m}^2/\text{W}$ and $n_4 = -6.0 \times 10^{-32} \text{ m}^4/\text{W}^2$. In general, the nonlinear coefficients γ_i ($i = 1, 2$) can be estimated from $\gamma_i = \frac{2\pi n_j}{\lambda A_{eff}^i}$, where n_j ($j = 2, 4$) are the nonlinear refractive index coefficient. $A_{eff} = \pi w^2$ is the effective fiber core area, with w is the core radius of the fiber, which varies from (3–3.8) μm for As_2Se_3 and (1.3–1.7) μm for the As_2S_3 . λ , the typical telecommunication wavelength 1.55 μm . For the range of $|\gamma_1| = (2000 - 3000) \text{ W}^{-1}/\text{Km}$, cubic nonlinear coefficient, we have calculated the values for quintic nonlinear coefficients: $|\gamma_2| = (1.3 - 3) \text{ W}^{-2}/\text{Km}$ for As_2Se_3 and for $|\gamma_2| = (3.3 - 7.6) \text{ W}^{-2}/\text{Km}$ for As_2S_3 respectively. The 1.55 μm window today is mainly of interest to long-distance telecommunications application.

The other important feature of this transmitted wavelength is that it matches the fiber's low-loss regions. Fiber energy loss (absorption) compensation with sufficient Raman gain and distortionless propagation of picosecond soliton pulses in a monomodal optical fiber have been experimentally demonstrated by Mollenauer et. al. [14]. Chalcogenide glasses have attracted much interest in the past few years as a nonlinear optical material in the telecommunications wavelength window of 1550 nm, and are promising candidates for planar non-linear optical (NLO) rib waveguide devices due to high nonlinearity, high refractive index, and non-linear optical losses (0.05 dB/cm) at 1550 nm [15, 16]. In a very recent work, M. El-Amraoui et. al. [17] measured the fiber losses as low as 0.35 dB/m at 1.55 μm for a 45 meters long 2.3 m core size fiber. The related nonlinear Kerr coefficient is estimated as high as $2750 \text{ W}^{-1}\text{km}^{-1}$. It is also an important to note the fact that chalcogenide glasses exhibit the highest nonlinear refractive indices and suffer, at worst, only moderately from two-photon absorption, also they do not suffer from free-carrier absorption. The nonlinear absorption faced by the fiber material can be compensated using derivative higher-order nonlinear Raman gain terms. In this context, Tuniz et. al. [18] studied how Raman gain and nonlinear absorption counteract across the C and L-bands in two-photon absorption effects in single-mode chalcogenide fiber.

Now, physically, for the ultrashort laser pulse propagation through optical fiber at telecommunication wavelength 1.55 μm , (last reference of [2]) and carrier frequency $\omega_0 = 1.22 \times 10^{15} \text{ s}^{-1}$ i.e. $T_0 = 5.1475 \times 10^{-15} \text{ s}$, if we choose the typical real experimental value for the model parameters of Eq. (2) as $a_1 = \frac{\beta_2}{2} = 10 \text{ ps}^2/\text{km}$, $a_2 = \gamma_1 = 2765 \text{ W}^{-1}/\text{km}$, $a_3 = \frac{\beta_3}{6} = 0.0235 \text{ ps}^3/\text{km}$, $a_4 = -\frac{\gamma_1}{\omega_0} = -14.2328 \text{ W}^{-1}/((2\pi)\text{km THz})$ and $a_5 = \gamma_1 T_R = 14931 \text{ W}^{-1} \text{ fs}/\text{km}$ ($T_R = 5.4 \text{ fs}$ for chalcogenide glass fiber [13]), we can estimate the size of the coefficients of the non-Kerr nonlinearities of Eq. (2) from the constraint relations in Eq.(6). The calculated values for the coefficients of non-Kerr nonlinearities are $a_6 = \gamma_2 = \frac{3a_2^2 a_3}{5a_1^3} = 2.533 \text{ W}^{-2}/\text{km}$, $a_7 = \frac{3a_2 a_3^2 a_4}{5a_1^3} = -1.304 \times 10^{-2} \text{ W}^{-2}/((2\pi)\text{km THz})$ and $a_8 = \frac{a_2 a_3^2 a_5}{2a_1^3} = 11.3996 \text{ W}^{-2} \text{ fs}/\text{km}$. For the sub-10fs bright pulse communication in non-Kerr medium with the above model parameter values for Eq. (2), one can check that we need chalcogenide optical fiber of the type As_2Se_3 of core radius 3.20415 μm and for As_2S_3 , the core radius is 1.399 μm respectively. If we use the same parameter values for the dark pulse propagation, one can obtain the energy value for dark pulse will be $\mathcal{E} = -\frac{a^2}{6b} = -\frac{(v-2\Omega+3\Omega^2)^2}{3\alpha_1+2\alpha_2} = -88.779$.

In conclusion, we have reported optical bright and dark solitary wave solutions of Higher-Order Nonlinear Schrödinger equation in presence of non-Kerr terms subject to constraint relations among the parameters. The derivative Kerr and non Kerr nonlinear terms are

important for compensation of the nonlinear absorption during propagation in highly nonlinear material. These terms also play an important role for the post-soliton compression to get highly stable compressed optical pulse. The functional forms of bright and dark solitary wave profiles reported are new. We also have presented the periodic solutions which are very meaningful in optics. We have seen that the estimated values for non-Kerr nonlinear model parameters of Eq. (2) agreed the reality of the waveguide made of chalcogenide glasses. So the inclusion of the non-Kerr nonlinear terms in Eq. (1) is justified to describe the sub-10fs pulse propagation in highly nonlinear optical fiber. As well as Case 1, we have checked the parametric condition in Eq. (14) for Case 2 using the model parameters given above. The quintic non-Kerr nonlinear terms in contemporary optics become very crucial to the upcoming applications in ultrafast signal routing systems, double doped optical fiber, optical switching etc. The

periodic solution can be used to study the formation of solitons in the periodic structure if one consider the quintic non-Kerr nonlinearity in fiber Bragg grating [19], and finally, with the calculated parameter values for highly nonlinear optical fiber made of chalcogenide glasses, the HNLS equation, in presence of non-Kerr nonlinear terms as higher-order perturbation, not only could find application in broadband telecommunication that extends far into the infrared (IR) spectral region for the bright and dark optical pulses but also Eq. (2) may be a new theoretical model equation for experimental designing sub-10fs optical pulse propagation, which will be applicable for the next generation optical fiber using chalcogenide type high nonlinear optical glasses.

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